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Approximate and Conditional Teleportation of an Unknown Atomic State Without the Bell-state Measurement with Multi-photon Interaction

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A scheme for approximately and conditionally teleporting an unknown atomic state via multi-photon interaction in cavity quantum electro dynamics (QED) is proposed. It is the extension of the scheme of Zheng (2004) [*Physical Review A69, 064302*], which is based on Jaynes–Cummings model in QED and where only a time point of system evolution and the corresponding Fidelity implementing the teleportation are given. In our scheme, the cavity field may be Fock state and the multi-photon interaction Jaynes–Cummings model is used to realize the approximate and conditional teleportation. Our scheme does not involve the Bell-state measurement and an additional atom, only requiring two atoms and one single-mode cavity. The fidelity of the scheme is higher than that of Zheng (2004) [*Physical Review A69, 064302*]. The scheme may be generalized to not only the teleportation of the state of a cavity mode to another mode by means of a single atom but also the teleportation of the state of a trapped ion.

KEY WORDS: teleportation; bell-state measurement; cavity QED; multi-photon interaction.
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Quantum entanglement is one of the most striking features of quantum mechanics and plays an key role in quantum information processing. When two subsystems are entangled, the whole state vector cannot be separated into a product of the states of the subsystems and thus the two subsystems are no longer independent even if they are far spatially separated. A measurement on one subsystem not only gives information about the other subsystem, but also provides possibilities of manipulating it. Bennett *et al.* (1993) demonstrated that the quantum entanglement can be utilized to teleport an unknown quantum state. At the beginning of the teleportation process, two spin-1/2 particles are prepared in the maximally entangled

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state, that is, Bell state. Then, a joint measurement is performed on the particle to be teleported and one of the entangled pair. In the end, the information of the joint measurement is sent to the other observer through a classical channel and thus he can reproduce the initial state of the teleported particle on the second particle of the entangled pair. Quantum teleportation has been experimentally demonstrated using optical systems (Bouwmeester *et al.*, 1997; Furusawa *et al.*, 1998; Boschi *et al.*, 1998), NMR (Nielsen *et al.*, 1998), and trapped ions systems (Riebe *et al.*, 2004; Barrett *et al.*, 2004; Leibfried *et al.*, 2003).

On the other hand, the cavity quantum electro dynamics (QED) system is another qualified candidate for demonstrating quantum information processing. Cirac and Parkins (1994) have proposed a scheme for the realization of quantum teleportation of an unknown atomic state by using two additional atomic levels of the entangled pair in cavity QED. Davidovich *et al.* (1994) have presented another proposal for the teleportation of an unknown atomic state between two cavities initially prepared in an entangled photon-number states. Other unconditional schemes, none of which have been experimentally implemented, have also been proposed for atomic state teleportation in cavity QED (Bose *et al.*, 1995; Zheng and Guo, 1997; Zheng, 1999). Fang and Cao (2004a,b, 2005a,b) have also presented some schemes for the teleportation of multi-particle state with the Bell-measurement. Recently, Zheng (2004) has proposed a novel scheme for the approximate conditional teleportation of an unknown atomic state in cavity QED, the distinct feature of which is that it does not require the Bell-state measurement on two qubits. Thus, an additional atom is unnecessary and the scheme involve only two atoms interacting with a single-mode cavity field. However, only a time point of system evolution and the corresponding fidelity implementing the teleportation are given in the scheme. In this paper, we propose another novel scheme, which is based on the multi-photon interaction Jaynes–Cummings model in the cavity QED, and then calculate multi-time points and the corresponding fidelities and then use them to realize the approximate and conditional teleportation of an unknown atomic state. Naturally, our scheme does also not involve the Bell-state measurement and an additional atom, only requiring two atoms and one single-mode cavity. The fidelity of the scheme is higher than that of Zheng (2004). Atom *b,* which receives the teleported state, is first entangled with the cavity mode. Then, atom *a*, whose state is to be teleported, interacts with the cavity mode. A measurement on atom *a* may directly and approximately collapse atom *b* to the initial state of atom *a* with the cavity mode left in the vacuum state. The scheme may be generalized to not only the teleportation of the state of a cavity mode to another mode by means of a single atom but also the teleportation of the state of a trapped ion.

In the interaction picture, the atom-cavity resonant interaction is described by the Jaynes–Cummings Hamiltonian of two-photon interaction

$$H_i = \lambda(a^{+l} S^- + a^l S^+), \tag{1}$$

where a^+ and a are the creation and annihilation operators for cavity field, S^+ and S^- are the raising and lowering operators for atom in the cavity, λ is the atom-field coupling constant. We assume that atom a to be teleported is initially prepared in the state

$$|\psi\rangle_a = c_g\, |g\rangle_a + c_e\, |e\rangle_a\,, \tag{2}$$

where $|g\rangle_a$ and $|e\rangle_a$ are the ground and excited states of the atom, with c_g and c_e being unknown coeffieients. Atom b, to receive the teleported state, is initially prepared in the state $|e\rangle_b$. We send atom b through an initially empty resonant cavity. After interaction time t, the atom exits the cavity and thus the state of the system becomes into

$$\begin{aligned} |\phi\rangle = {} & \cos\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t\right) |e\rangle_b\, |n\rangle \\ & - i \sin\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t\right) |g\rangle_b\, |n+l\rangle\,, \end{aligned} \tag{3}$$

The atomic velocity is carefully chosen so that $\lambda t = \pi/(4\sqrt{\frac{(n+l)!}{n!}})$ is fulfilled. Then we can obtain the maximally entangled state between the atomic internal states and the cavity modes

$$|\phi\rangle = \frac{1}{\sqrt{2}}[|e\rangle_b\, |n\rangle - i\, |g\rangle_b\, |n+l\rangle]. \tag{4}$$

Now, the whole system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}[c_g\, |g\rangle_a + c_e\, |e\rangle_a][|e\rangle_b\, |n\rangle - i\, |g\rangle_b\, |n+l\rangle]. \tag{5}$$

Then let atom a interact with the cavity mode. After an interaction time t', the system evolves into

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\left\{c_g\, |g\rangle_a\, |e\rangle_b\, |n\rangle - i c_g\, |g\rangle_b \left[\cos\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right) |g\rangle_a\, |n+l\rangle\right.\right.$$

$$- i\sin\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right)|e\rangle_a|n\rangle\Bigg] + c_e|e\rangle_b\Bigg[\cos\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right)|e\rangle_a|n\rangle$$

$$- i\sin\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right)|g\rangle_a|n+l\rangle\Bigg] - ic_e|g\rangle_b$$

$$\times\Bigg[\cos\left(\sqrt{\frac{(n+2l)!}{n!}}\lambda t'\right)|e\rangle_a|n+l\rangle$$

$$- i\sin\left(\sqrt{\frac{(n+2l)!}{n!}}\lambda t'\right)|g\rangle_a|n+2l\rangle\Bigg]\Bigg\} \tag{6}$$

The detection of atom a in the state $|e\rangle_a$ collapses the system consisting of atom b and the cavity field to

$$|\psi''\rangle = N[-c_g\sin(\sqrt{2}\lambda t')|g\rangle_b|0\rangle + c_e\cos(\sqrt{2}\lambda t') \times |e\rangle_b|0\rangle - ic_e\cos(2\sqrt{3}\lambda t')|g\rangle_b|4\rangle], \tag{7}$$

where N is a normalization factor. When we choose the atomic velocity carefully so that $\sqrt{\frac{(n+l)!}{n!}}\lambda t' = 2k\pi - \pi/4$, with k being integer, and thus $|\cos(\sqrt{\frac{(n+2l)!}{n!}}\lambda t')| \le 0.05$. Then the atom b is approximately in the initial state of the atom a,

$$|\psi\rangle_b = c_g|g\rangle_b + c_e|e\rangle_b\,, \tag{8}$$

with the cavity mode left in the Fock state $|n\rangle$. The fidelity is

$$F = \frac{1/2}{1/2 + |c_e|^2\cos^2(\sqrt{\frac{(n+2l)!}{n!}}\lambda t')} \ge 0.995. \tag{9}$$

Through complex computation, we find that when the evolution time is as follows

$$\sqrt{\frac{(n+l)!}{n!}}\lambda t' = \left(2k - \frac{1}{4}\right)\pi, \quad k = 0, 1, \ldots, \tag{10}$$

$\cos(\sqrt{\frac{(n+2l)!}{n!}}\lambda t') \approx 0$, then the fidelity $F \approx 1$. For example, if $l = 2$ and $n = 0$, $\sqrt{\frac{(n+l)!}{n!}}\lambda t' = (20k + \frac{71}{4})\pi$, $k = 0, 1, \ldots$, $\cos(\sqrt{\frac{(n+2l)!}{n!}}\lambda t') = \pm 0.03962 \approx 0$, and the fidelity $F \ge 0.997$ being higher than that of Zheng (2004), whose value is 0.987. It is clear that we can still calculate many distinct time-evolution points, at which the approximate and conditional teleportation can be implemented. The probability of success is 0.25. The distinct advantage of the scheme is not only that the Bell-state measurement is not required but also that many distinct time-evolution points realizing the conditional teleportation can be given. A direct measurement on atom a may straightly and approximately project atom b on the initial state of atom a with the cavity mode left in the vacuum state. Naturally, the scheme is much simpler than the previous schemes, which require a third atom to preform the Bell-state measurement. At the meantime, the scheme has much richer context than that in Zheng (2004).

It is interesting that the idea can also be used to teleport the state of a cavity mode to another cavity mode using a single atom. We assume that the atom is initially prepared in the state $|e\rangle$. We first send this atom through an initially resonant cavity b in Fock state $|n\rangle$, and then choose the atomic velocity carefully so that $\lambda t = \pi/(4\sqrt{\frac{(n+l)!}{n!}})$ is fulfilled. Then we have

$$|\varphi\rangle = \frac{1}{\sqrt{2}}[|e\rangle\,|n\rangle_b - i\,|g\rangle\,|n+l\rangle_b]. \tag{11}$$

Assume that the cavity (cavity a) to be teleported is initially prepared in the state

$$|\phi\rangle_a = c_n\,|n\rangle_a + c_{n+l}\,|n+l\rangle_a\,, \tag{12}$$

where c_0 and c_1 are unknown coefficients. Now, the whole system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}[c_n\,|n\rangle_a + c_{n+l}\,|n+l\rangle_a][|e\rangle\,|n\rangle_b - i\,|g\rangle\,|n+l\rangle_b]. \tag{13}$$

Then let the cavity a interact with the atom. The system evolves to

$$\begin{aligned}
|\psi'\rangle = \frac{1}{\sqrt{2}}\Bigg\{ & - i c_n\,|n\rangle_a\,|g\rangle\,|n+l\rangle_b - i c_{n+l}\,|n+l\rangle_b \\
& \times\left[\cos\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right)|g\rangle\,|n+l\rangle_a - i\sin\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right)|e\rangle\,|n+l\rangle_a\right] \\
& + c_n\,|n\rangle_b\left[\cos\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right)|e\rangle\,|n\rangle_a - i\sin(\sqrt{\frac{(n+l)!}{n!}}\lambda t')\,|g\rangle\,|n+l\rangle_a\right] \\
& + c_{n+l}\,|n\rangle_b \times\left[\cos\left(\sqrt{\frac{(n+2l)!}{n!}}\lambda t'\right)|e\rangle\,|n+l\rangle_a\right. \\
& \left.\left. - i\sin\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right)|g\rangle\,|n+2l\rangle_a\right]\right\},
\end{aligned} \tag{14}$$

where t' is the interaction time between the atom and the cavity mode a. The detection of the atom in the state $|e\rangle$ project the system combined the cavity b and the cavity a to

$$|\Psi\rangle = N\left[c_n \cos\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right)|n\rangle_a\,|n\rangle_b - c_{n+l}\sin\left(\sqrt{\frac{(n+l)!}{n!}}\lambda t'\right)\right.$$
$$\left.\times\,|n\rangle_a\,|n+l\rangle_b + c_1\cos\left(\sqrt{\frac{(n+2l)!}{n!}}3\lambda t'\right)|n+2l\rangle_a\,|n\rangle_b\right], \qquad (15)$$

where N is a normalization factor. When $\sqrt{\frac{(n+l)!}{n!}}\lambda t' = 2k\pi - \frac{\pi}{4}, k = 0, 1, 2, \ldots$ and $\cos(\sqrt{\frac{(n+2l)!}{n!}}\lambda t') \simeq 0$ are fulfilled, cavity b is approximately in the state of the cavity a.

$$|\phi\rangle_b = c_n\,|n\rangle_b + c_{n+l}\,|n+l\rangle_b\,, \qquad (16)$$

with cavity a left in the Fock state $|n\rangle$. In the case of $l = 2$ and $n = 0$, $\sqrt{\frac{(n+l)!}{n!}}\lambda t' = (20k + \frac{71}{4})\pi, k = 0, 1, \ldots,$ $\cos(\sqrt{\frac{(n+2l)!}{n!}}\lambda t') = \pm 0.03962 \approx 0,$ the fidelity $F \geq 0.997$. Therefore, the approximate teleportation of a cavity mode to another cavity mode can easily be realized in the two-photon interaction in QED. Naturally, in our case only a single atom is required.

Now, we turn to discuss the experimental feasibility of the proposed scheme. We assume that the cavity field is vacuum state and the two-photon interaction is used in our scheme, i.e., $l = 2$. For the Rydberg atoms with principal quantum numbers 50 and 51, the radiative time is $T_r = 3.5 \times 10^{-2}s$, and the coupling constant is $\lambda = 2\pi \times 25$ Hz (Rauschenbeutel *et al.*, 2001). Thus, the interaction time of the atoms b and a with the cavity field are $\pi/(4\sqrt{2}\lambda) = 0.35 \times 10^{-5}$s, $71\pi/(4\sqrt{2}\lambda) = 71 \times 0.35 \times 10^{-5}$ s, respectively. Then we assume that the traveling times of these atoms are 0.35×10^{-5} s and $71 \times 0.35 \times 10^{-5}$, respectively. Thus, the time required to complete the whole procedure is about 2.52×10^{-4} s, much shorter than T_r. The decay time of the cavity is of the order $T_c \simeq 2.0 \times 10^{-3}$ s, longer than the required time. Therefore, based on the QED techniques presently or soon, the proposed scheme might be implemented.

In fact, the idea can also be generalized to the teleportation of an unknown state of an ion to another ion in a linear trap. Two two-level ions confined in such a linear trap are initially prepared in the vibrational ground state by laser. Ion a, to be teleported, is initially in the electronic state

$$|\psi\rangle_a = c_g\,|g\rangle_a + c_e\,|e\rangle_a\,, \qquad (17)$$

Ion b is initially prepared in the electronic excited state $|e\rangle_b$. The second ion is driven by a laser tuned to the second vibrational sideband. In the Lamb-Dicke limit, if the phase of the laser field is adjusted to zero, the Hamiltonian is (Cirac and Zoller, 1995; Zeng *et al.*, 2004, 2005; Chen and Gao, 2005)

$$H = \eta\Omega(a^{+l}S^- + a^l S^+), \tag{18}$$

where a^+ and a are the creation and annihilation operators of the collective motion of the trapped ions, and Ω is the Rabi frequency. The Hamiltonian has the same form of (1) with $\lambda = \eta\Omega$. After an interaction time $\pi/(4\sqrt{\frac{(n+l)!}{n!}}\eta\Omega)$, ion b and the collective motion is prepared in the maximally entangled state. Then ion a is excited by a laser, also tuned to the second lower vibrational sideband. After an interaction times $t' = (20m + \frac{71}{4})\pi/(\sqrt{2}\eta\Omega)$, $m = 0, 1, \ldots$, in the case of $l = 2$ and $n = 0$, the detection of ion a in the $|e\rangle_a$ approximately projects the ion b into the initial electronic state of ion a.

In summary, we have proposed a scheme for approximately and conditionally teleporting an unknown atomic state in cavity QED. Our scheme is based on the multi-photon interaction Jaynes–Cummings model. It is the extension of the scheme of Zheng (2004), where only a time point of system evolution and the corresponding fidelity implementing the teleportation are given. We have given the general formulae of time points of the system evolution and the corresponding fidelities, and then used them to realize the approximate and conditional teleportation. The fidelity F is than that of Zheng (2004). Naturally, our scheme does not involve the Bell-state measurement and an additional atom, only requiring two atoms and one single-mode cavity. The scheme has been generalized to not only the teleportation of the state of a cavity mode to another mode by means of a single atom but also the teleportation of the state of a trapped ion. Based on the QED techniques presently or soon, the proposed scheme might be implemented.

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